Kirchhoff's Rules

Objectives

After going through this lesson, learner will be able to:

- InterpretingKirchhoff's Rules
- Apply Kirchhoff's rules for calculating current and voltages in simple electrical circuits

Content Outline

- Unit Syllabus
- Module wise distribution of unit syllabus
- Words you should know
- Introduction
- Kirchhoff's rules
- Problem solving using Kirchhoff's rules for electrical circuits
- Application of Kirchhoff's rules
- Solved Examples
- Problems for practice
- Summary

Unit Syllabus

Electric current, flow of electric charges in a metallic conductor, drift velocity and mobility, and their relation with electric current; Ohm's law' electrical resistance, V-I characteristics (linear & non- linear), electrical energy and power, electrical resistivity and conductivity.

Carbon resistors, colour code for carbon resistors; series and parallel combinations of resistors; temperature dependence of resistance

Internal resistance of a cell, potential difference and emf of cell, combination of cells in series and in parallel.

Kirchhoff's laws and simple applications; Wheatstone bridge, Meter bridge.

Potentiometer- principle and its applications to measure potential difference & for comparing emf of two cells; measurement of internal resistance of a cell.

Module Wise Distribution

The above unit has been divided into 8 modules for better understanding.

Module 1	• Electric current, • Solids liquids and gases • Need for charge carriers speed of charge carriers in a metallic conductor • Flow of electric charges in a metallic conductor • Drift velocity, • Mobility and their relation with electric current • Ohm's law,
Module 2	• Electrical resistance, • V-I characteristics (linear and nonlinear), • Electrical energy and power, • Electrical resistivity and conductivity • Temperature dependence of resistance
Module 3	• Carbon resistors, • Colour code for carbon resistors; • Metallic Wire resistances • Series and parallel combinations of resistors • Grouping of resistances • Current and potential differences in series and parallel circuits
Module 4	• Internal resistance of a cell, • Potential difference and emf of a cell, • Combination of cells in series and in parallel. • Need for combination of cells
Module 5	• Kirchhoff's Rules • Simple applications. of Kirchhoff's Rules for calculating current s and voltages • Numerical
Module 6	• Wheatstone bridge • Balanced Wheatstone bridge condition derivation using Kirchhoff's Rules • Wheatstone bridge and Meter Bridge. • Application of meter bridge
Module 7	• Potentiometer –

	● Principle ● Applications to ● Measure potential difference ● Comparing emf of two cells; ● Measurement of internal resistance of a cell. ● Numerical
Module 8	● Numerical ● Electrical energy and power

Module 5

Words You Must Know

- **Potential drop across resistance:** It is the potential difference between ends of a resistance. In Ohm's law V = IR where, V is potential drop across resistance (for details see role of resistance in module 1)

- **Electromotive Force (EMF):** It is the force which makes change to flow in an electrical circuit. It is defined as work done in moving a unit positive charge once in a closed circuit.
- **EMF of a cell** (ε): It is the max. Potential difference between electrodes of a cell when no_current is being drawn from the cell.

- **Terminal potential difference of a cell (V)**: It is the max. Potential difference between electrodes of a cell when current is being drawn.

- **Internal resistance of a cell(r)**: It is the resistance offered by electrolyte to current flowing.
- **Series combination of resistances:** When the same current flows through all the devices, it is said to be a series combination.

- **Parallel combination of resistances:** When all devices have the same potential difference, it is said to be a parallel combination.

- **Electrical circuit:** It is the arrangement of electrical devices like resistance, cell, etc. to achieve a purpose or objective.

Introduction

Electric circuits generally consist of a number of resistors and cells interconnected sometimes in a complicated way. Have you ever wondered how we can find potential drop across a resistor or current in the arm of the circuit in the simplest manner? The formulae we have derived earlier for series and parallel combinations of resistors are not always sufficient to determine all the currents and potential differences in the circuit.

Two rules, called Kirchhoff's rules, are very useful for analysis of electric circuits.

Kirchhoff's Rules

These rules are very fine tools to analyze an electrical circuit. That is to find potential drop across a device or current flowing through an arm of a circuit etc.

First rule: - Junction or Current rule (KCL- Kirchhoff's current law)

It states that the algebraic sum of currents at a junction of an electrical circuit is zero.

or $\Sigma I = 0$

Sum of incoming currents = sum of outgoing currents

- **It is based on the law of conservation of charge.**
- **This rule is valid for both open and closed circuits.**

Second rule: - Kirchhoff's Voltage Rule (KVL- Kirchhoff's voltage law)

It states that in a closed electrical circuit the algebraic sum of EMF's of cells is equal to the algebraic sum of potential drops across resistances.

$\Sigma E = \Sigma IR$

- **This law is based on the law of conservation of energy.**
- **This law is valid only for closed circuits.**

Sign Conventions: - Since we are saying algebraic sum of emf and potential drops, there must be a sign convention to mark emf and potential drops as positive and negative.

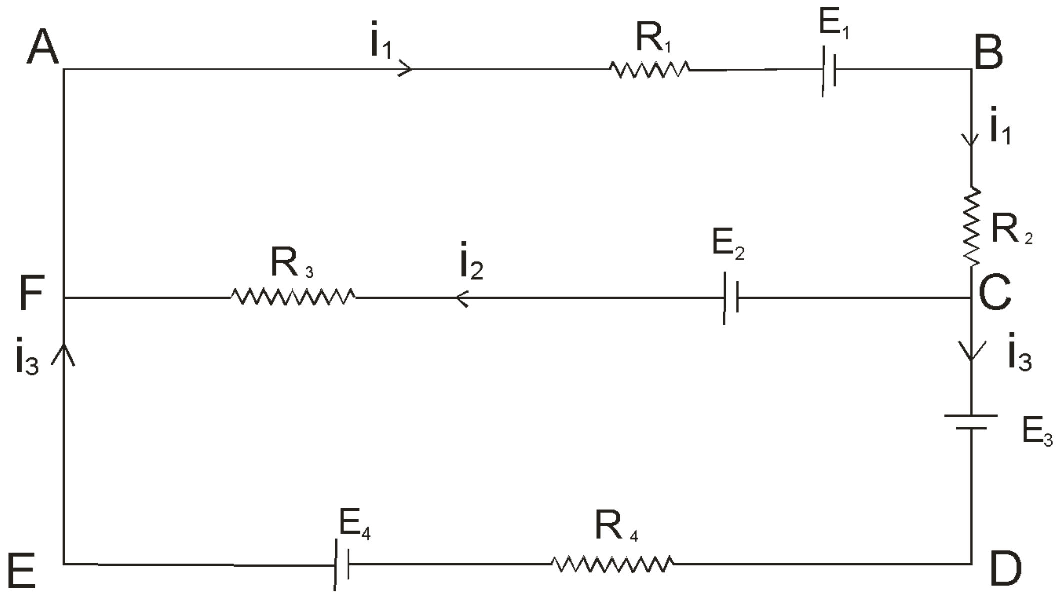
Here is the sign convention for emf and potential drops:

- **For EMF's of cells: if we go from '+' to '-'terminal of the cell then, emf is taken as negative (-ve). If we go from '-'to '+' terminal of the cell then, emf is taken as positive (+ve).**

- **For potential drops across resistances: If we go in the direction of current the potential drop is taken as negative (-ve). If we go in the direction opposite to current, the potential drop is taken as positive (+ve).**

Problem Solving using Kirchhoff's Rules

Let us take an example to understand Kirchhoff's Rules and applications of sign conventions.



For applying Kirchhoff's Rule following steps are to be taken:

- Draw a large circuit diagram so that there is enough space for labeling.
- Label each junction along with few other points as A, B, C….etc for the identification of various closed parts i.e. loops of the circuit.
- Label all quantities i.e. resistances, currents and emfs. Assume the direction of each unknown current based on the fact that a current from a source starts from its positive terminal and flows through the external circuit to reach its negative terminal.

  If the actual direction of the current is opposite to what we have assumed, the calculated value of the current will have a negative sign. Thus, correct use of Kirchhoff's laws not only gives us the magnitude but also the directions of unknown currents.

- Use junction rules to express the currents that reach or leave the junctions. This helps in minimizing the number of unknown currents in the circuit and hence reduces the calculation work.
- Choose any closed loop of the circuit and travel around it to finally return back to the point of start. While doing so, write the potential differences across resistors i.e. I × R and emf with proper sign i.e. (+)ve or (-)ve. Use Kirchhoff's laws and equate their sum to zero to get an independent equation.
- Choose more loops to get as many independent equations as the unknown currents.
- Solve these equations simultaneously to find the unknown quantities.

Let us now apply Kirchhoff's rules and sign conventions.

As per Kirchhoff's First rule, at junction C&F

$$I_1 = I_2 + I_3$$

In loop ABCDEFA

$$-I_1R_1 - E_1 - I_1R_2 - E_3 - I_3R_4 + E_4 = 0$$

In loop ABCFA

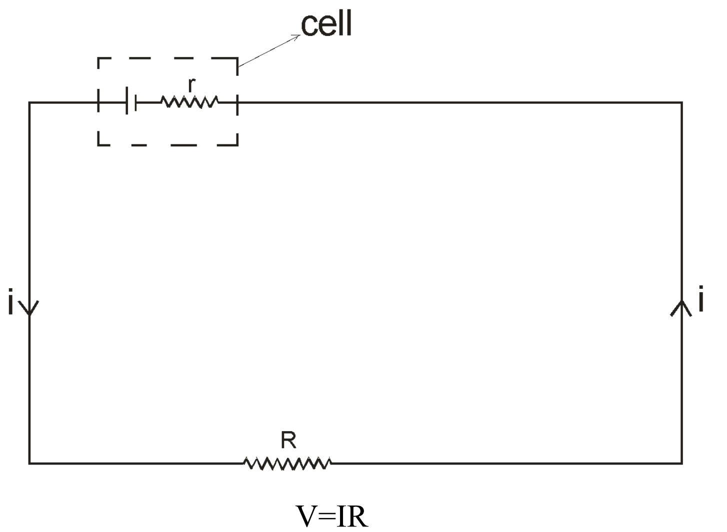
$$-I_1R_1 - E_1 - I_1R_2 - E_2 - I_2R_3 = 0$$

In loop FCDEF

$$I_2R_3 + E_2 - E_3 - I_3R_4 + E_4 = 0$$

Applications of Kirchhoff's Rules

a) Relation between emf (e), TPD (v) and internal resistance of cell (r)



V=IR

By applying Kirchhoff's Voltage rule

$-IR - Ir + E = 0$

$-V - Ir + E = 0$ . . . . for discharging of cell

E>V $V = E - Ir$

The above equation is for discharging of cells. Note that in discharging of cell emf > TPD

Equation for charging of cell is given by

$V = E + Ir$

$V > E$ . . . . for charging of cell

For charging of cell TPD > emf

Expression for internal resistance(r) of cell

From $V = E - Ir$

$Ir = E - V$

$$r = \left(\frac{E-V}{I}\right)R$$

b) Combinations of Cells

i. Cells in series

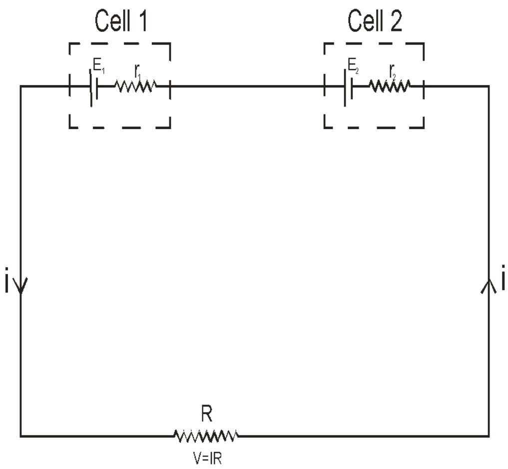


Fig: a showing E_1 and E_2 having

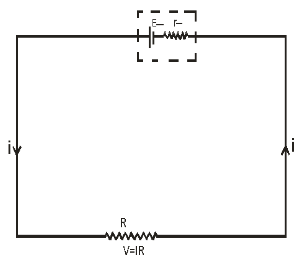


Fig: b showing E_{eq} & r_{eq}

Internal resistances r_1 and r_2

By Kirchhoff's Voltage rule

$-IR - Ir_2 + E_2 - Ir_1 + E_1 = 0$

$-V + (E_1 + E_2) - I(r_1 + r_2) = 0$

$V = (E_1 + E_2) - I(r_1 + r_2)$

Comparing it with $V = E_{eq} - Ir_{eq}$

$E_{eq} = E_1 + E_2$

$r_{eq} = r_1 + r_2$

Why should we compare it with $V = E_{eq} - Ir_{eq}$?

It is because the two cells E_1 and E_2 combine to form an equivalent cell (E_{eq}), discharging equation of which is given by

$$V = E_{eq} - Ir_{eq}$$

If there are 'n' identical cells in series

$E_{eq} = nE$

$r_{eq} = nr$

$V = nE - Inr \Rightarrow IR = nE - nIr$

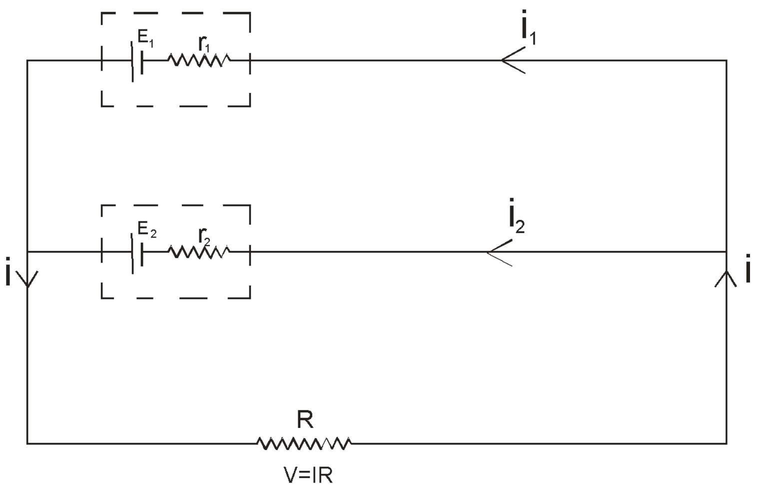
$I = \frac{nE}{R+nr}$

I will be max if R >> nr

$I_{max} = \frac{nE}{R}$

Max current can be drawn from a series combination, if external resistance is much greater than total internal resistance of cells.

ii. Cells in parallel



By Kirchhoff's junction rule,

At junctions C and F

$$I = I_1 + I_2$$

For loop ABCDEFA

$$-E_1 + I_1 r_1 + IR = 0$$

$$-E_1 + I_1 r_1 + V = 0$$

$$I_1 = \frac{E_1 - V}{r_1}$$

In loop FEDCF

$$-IR - I_2 r_2 + E_2 = 0$$

$$-V - I_2 r_2 + E_2 = 0$$

$$I_2 = \frac{E_2 - V}{r_2}$$

Putting I_1 and I_2 in (1)

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

$$I = \frac{E_1 r_2 - V r_2 + E_2 r_1 - V r_1}{r_1 r_2}$$

$$I r_1 r_2 = (E_1 r_2 + E_2 r_1) - V(r_1 + r_2)$$

Dividing equation by $(r_1 + r_2)$ to bring it in standard form of discharging equation

$$V = E - Ir$$

$$I\left(\frac{r_1 r_2}{r_1 + r_2}\right) = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - V$$

Comparing it with $V = E_{eq} - I r_{eq}$

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

$$\Rightarrow \quad \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

If there are n identical cells in parallel

$$E_{eq} = E \qquad (E_1 = E_2 = E,\ r_1 = r_2 = r)$$

$$\frac{1}{r_{eq}} = \frac{1}{r} + \frac{1}{r} + \ldots\ n\ times$$

$$r_{eq} = \frac{r}{n}$$

$$V = E_{eq} - Ir_{eq}$$

$$Ir = E - \frac{Ir}{n}$$

$$I = \frac{E}{R + \frac{r}{n}}$$

I will be max if $r/n >> R$

$$I_{max} = \frac{E}{\frac{r}{n}} = \frac{nE}{r}$$

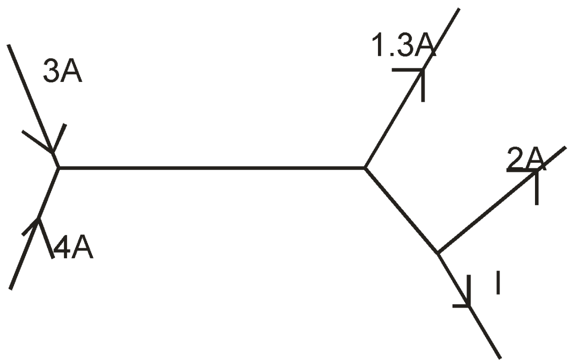
- Maximum current can be drawn from a parallel combination if total internal resistance is much greater than total external resistance.
- Max current can be drawn from a series combination, if external resistance is much greater than total internal resistance of cells.
- Maximum current can be drawn from a parallel combination if total internal resistance is much greater than total external resistance.
- Maximum power transfer theorem :-

Maximum power can be transferred from combinations of cell to the external resistance if external resistance is equal to total internal resistance of the cell i.e., $R = r_{eq}$

Solved Examples

Example

Find the value of current I in the circuit in the figure.

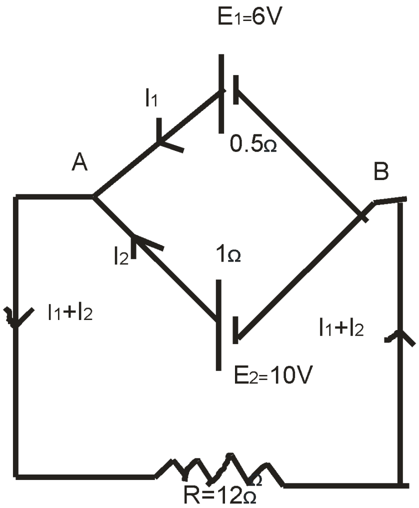


Solution

I= 3 + 4 – 1.3 – 2 = 3.7A

Example

A battery of 6V & internal resistance 0.5Ω is connected in parallel with another of 10V & internal resistance 1Ω. The combination sends a current through an external resistance of 12Ω. Find the current through each battery.



Solution

The arrangement of two batteries and external resistance is shown in the figure. Let I_1& I_2 be the currents given by the two batteries so that the current through the external resistance is (I_1+I_2) as shown in the figure.

Applying Kirchhoff's 2nd rule to the closed circuit $ARBE_1A$,

$$(I_1+I_2) \times 12 + I_1 \times 0.5 - 6 = 0$$

Or $$12.5I_1 + 12I_2 = 6 \quad \text{.......(i)}$$

Applying Kirchhoff's 2nd rule to the closed circuit $ARBE_2A$, we get

$$(I_1 + I_2) \times 12 + I_2 \times 1 - 10 = 0$$

Or $$12I_1 + 13I_2 = 10 \quad \text{........(ii)}$$

Multiplying (i) by 13 & (ii) 12, we get

$$162.5I_1 + 156I_2 = 78 \quad \text{.......(iii)}$$

$$144I_1 + 156I_2 = 120 \quad \text{.......(iv)}$$

Subtracting (iv) from (iii), we get

$$18.5I_1 = -42 \quad \text{or} \quad I_1 = -42/18.5 = -2.27\text{A}$$

Negative sign shows that I_1 actually flows in a direction opposite to what is shown in the figure

Substituting the value of I_1 in (ii),

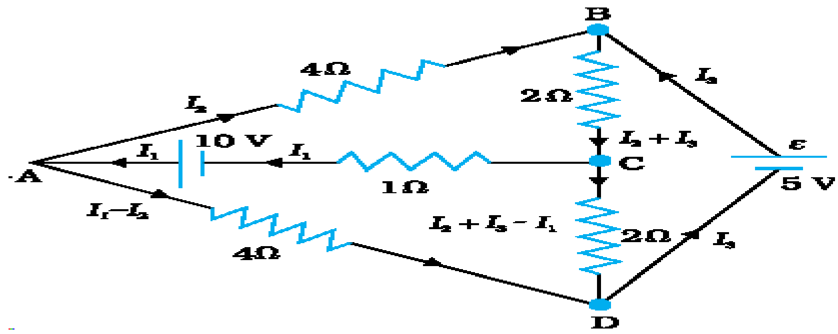
$$-12\times42/18.5 + 13I_2 = 10$$

Or $$13I_2 = 10+12\times42/18.5 = 185 + 504/18.5$$

$$I_2 = 689/18.5\times13 = 2.86 \text{ A}$$

Example

Determine current in each branch of the network shown in figure.



Solution

Each branch of the network is assigned an unknown current to be determined by the application of Kirchhoff's rules. To reduce the number of unknowns at the outset, the first rule of Kirchhoff is used at every junction to assign the unknown current in each branch. We then have the three unknowns I_1, I_2 and I_3 which can be found by applying the second rule of Kirchhoff to three different closed loops. Kirchhoff's second rule for the closed loop ADCA gives:

$$10 - (I_1 - I_3)4 - (I_1 + I_2 - I_3)2 - I_1 \text{ x } 1 = 0$$

Or

$$7I_1 + 2I_2 - v6I_3 = 10 \quad \ldots. \text{(i)}$$

In the closed circuit ADCA,

$$- I_3 \times 4 - (I_3 - I_2)2 - I_1 \times 1 + 10 = 0$$

Or

$$I_1 - 2I_2 + 6I_3 = 10 \quad \ldots. \text{(ii)}$$

In the closed current ABEDA

$$- (I_1 - I_3)\, 4 - 5 - I_3 \times 4 = 0$$

Or

$$4I_1 - 8I_3 = - 5 \quad \ldots. \text{(iii)}$$

On solving (i),(ii),(iii), we get

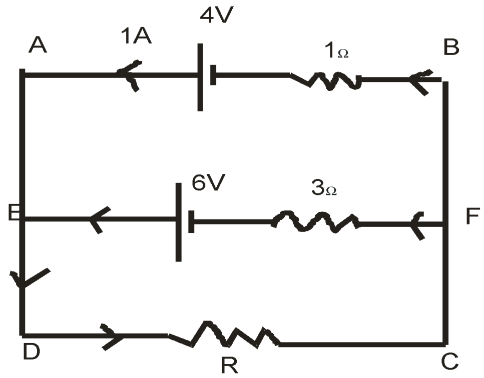
$$I_1 = 2.5A$$

$$I_2 = 1.875 \text{ A} = I_3$$

Example

Using Kirchhoff's rules, determine

i. the voltage drop across the unknown resistor R
ii. the current flowing in the arm EF in the circuit as shown:



Solution

i. Applying Kirchhoff's rule we get

$V_B - V_A = -1+4=3$, where V_A and V_B are potential at A and B

$V_B - V_A = 3$

Now, A is directly connected to B and is connected to C

Therefore, $V_A = V_D$ and $V_B = V_C$

$$V_D - V_C = 3 = V_F - V_E$$

Therefore, Potential difference across R is 3V

ii. Also, $V_F - V_E = -3I+6$

$$3 = -3I+6$$

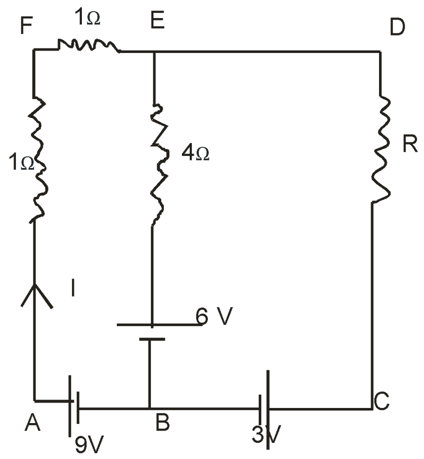
$$3I = 3$$

$$I = 1A$$

Therefore, current through EF is 1A

Example

Using Kirchhoff's rules determine the value of unknown resistor R in the circuit so that no current flows through 4Ω resistance. Also find the potential difference between A &D.



Solution

On applying Kirchhoff's voltage rule for loop FABEF

$$+2I - 9 + 6 + 4 \times 0 = 0$$

$$2I = 3$$

$$I = 1.5A$$

For loop BCDEB

$$3 + IR + 4 \times 0 - 6 = 0$$

$$IR = 3$$

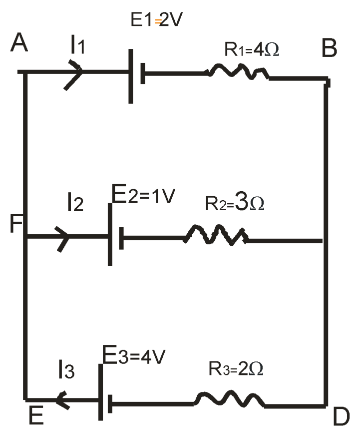
Substituting values of current I= 1.5A in the above equation, we get

$$R = \frac{3}{1.5}$$

$$R = 2\Omega$$

Problems for Practice

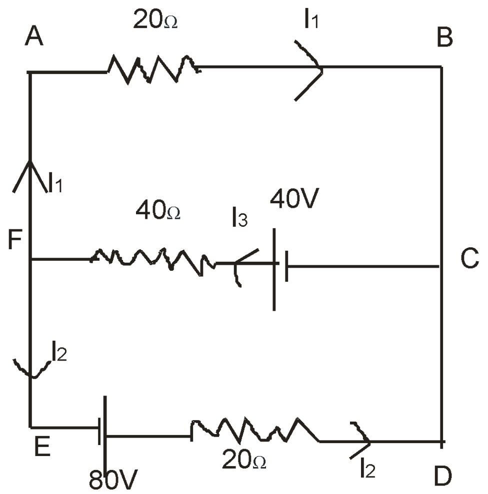
Question1: Using Kirchhoff's rules; write the expression for the currents I_1, I_2 and I_3 in the circuit diagram shown below.



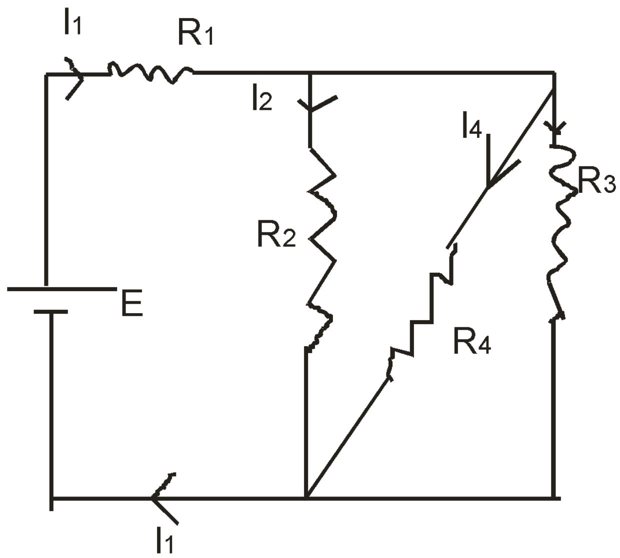
Question2: State and explain Kirchhoff's rule.

Question3: Are Kirchhoff's rules applicable to both a.c. and d.c.?

Question4: Using Kirchhoff's rules determine the value of current I_1 in the electric circuit given in figure.

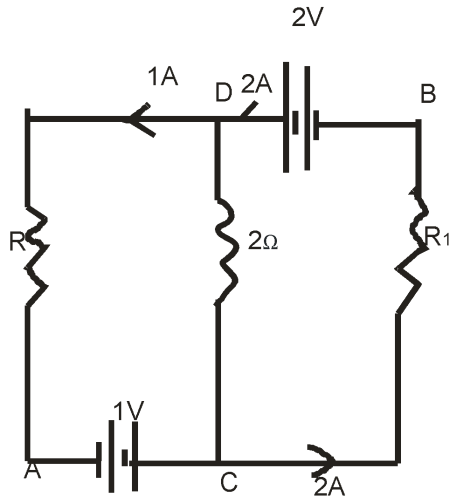


Question5: In the circuit shown in figure, $R_1 = 4\ \Omega$, $R2 = R_3 = 15\ \Omega$, $R_4 = 30\ \Omega$ and $E = 10$ V. Calculate the equivalent resistance of the circuit and current in each resistor.



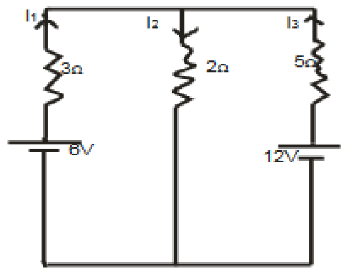
Question 6: State the fundamental concepts on which two Kirchhoff's rules are based.

Question7: In the given circuit, assuming point A to be at zero potential, use Kirchhoff's rules to determine potential at point B.

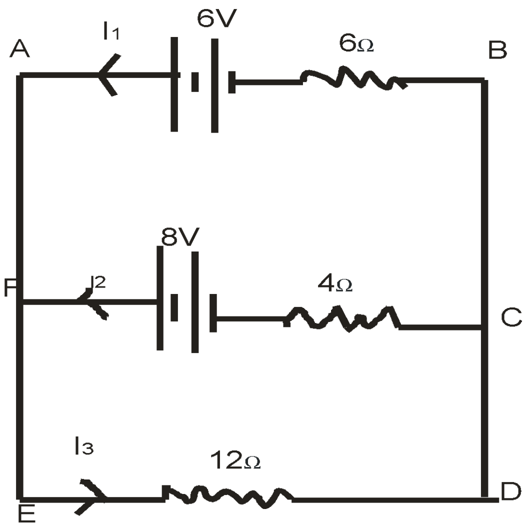


Question 8: State Kirchhoff's rules of current distribution in an electrical network.

Question 9: In the network given below use kirchhoff's rules to calculate the values of electric currents I_1, I_2 and I_3



Question10: In the network shown here, find the following:

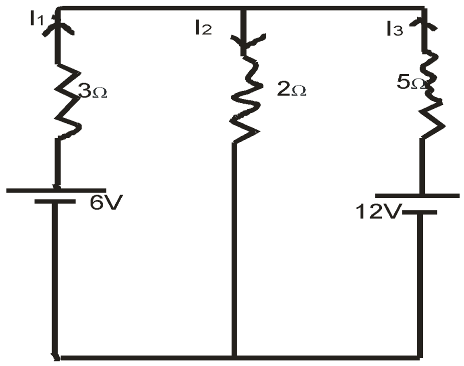


(a) Currents I_1, I_2 and I_3

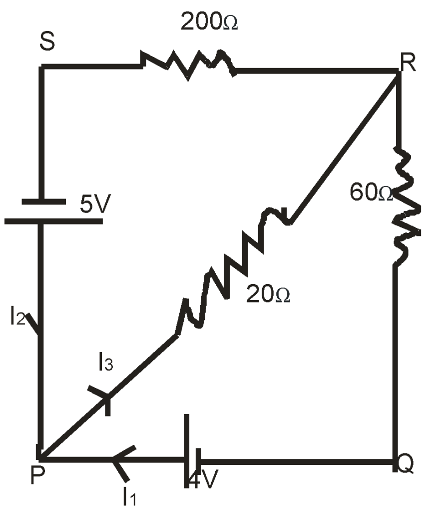
(b) TPD of each battery

Consider 6Ω to be internal resistance of 6V battery and 4Ω to be internal resistance of 8V battery.

Question11: In the network given below, use Kirchhoff's rules to calculate the values of electric currents I_1, I_2 and I_3.

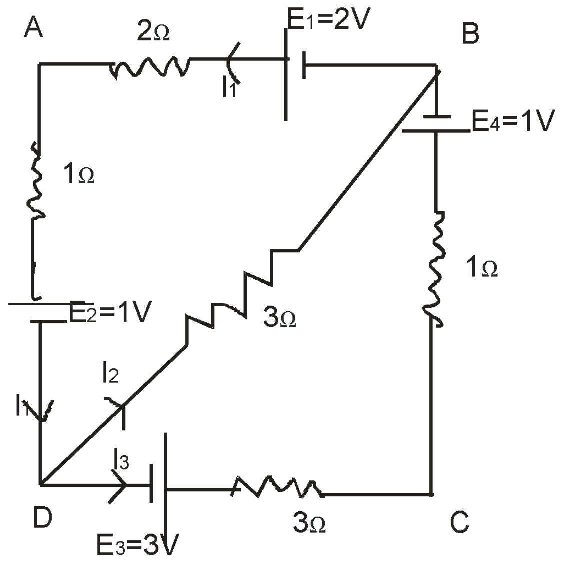


Question12: Apply these rules to the loops PRSP and PRQP to write the expressions for the currents I_1, I_2 and I3 in the given circuit.



Question 13: What is the significance of Kirchhoff's rules?

Question 14: In the given network, find the values of the currents I_1, I_2 and I_3.



Answers

1. $I_1 = \frac{2}{13}$ A , $I_2 = \frac{7}{13}$ A , $I_3 = \frac{9}{13}$ A
2. First rule: - Junction or Current rule (KCL- Kirchhoff's current law)

   It states that the algebraic sum of currents at a junction of an electrical circuit is zero.

   $\sum I = 0$

   or

   At a junction in the circuit, the sum of incoming currents = Sum of outgoing currents

   - It is based on the law of conservation of charge.
   - This rule is valid for both open and closed circuits.

   Second rule: - Kirchhoff's Voltage Rule (KVL- Kirchhoff's voltage law)

   It states that in a closed electrical circuit the algebraic sum of EMF's of cells is equal to the algebraic sum of potential drops across resistances. $\sum E = \sum IR$

3. Yes
4. $I_1 = -1.2$A
5. 10Ω, $I_1 = 1$A , $I_2 = I_3 = \frac{6}{15}$A, $I4 = \frac{6}{30}$A
6. Kirchhoff's first rule is based on the fact that the charges are not accumulated at a junction.

   Kirchhoff's second rule supports the law of conservation of energy.

7. $V_B = 1\text{volt}$

8. i) $I_1 = 0,\ I_2 = \frac{1}{2}A,\quad I_3 = \frac{1}{2}A$

ii) $V_{AB} = 6V,\quad V_{FC} = 6V$

9. $I_1 = \frac{18}{31}A,\ I_2 = \frac{66}{31}A,\ I_3 = \frac{48}{31}A$

10. $I_1 = \frac{39}{860}A,\quad I_2 = \frac{4}{215}A,\quad I_3 = \frac{11}{172}A$

11. $I_1 = \frac{13}{33}A,\quad I_2 = -\frac{2}{33}A,\quad I_3 = \frac{15}{33}A$

Summary

You have learnt in this module

- Potential drop is the difference in potentials across ends of a conductor.
- Electromotive force (EMF) is the force which drives charge in a closed electrical circuit cell, generators are sources of EMF.
- EMF is defined as work done in moving a unit +ve charge once in a closed circuit.
- EMF (E) of a cell is defined as maximum potential difference b/w electrodes of a cell when no current is being drawn from the cell.
- Terminal Potential difference (TPD) (V) of a cell is defined as maximum potential difference b/w electrodes when current is being drawn from the cell.
- Internal Resistance (r) of a cell is the obstruction offered by electrodes & electrolytes of a cell to current flowing through it.
- Kirchhoff's rules are the tools to analyze an electrical circuit. They enable us to find potential drop or current in a branch or a section of the circuit device.
- Kirchhoff's first rule is current rule or junction rule which states that at an electrical junction sum of incoming currents = sum of outgoing currents.
    - First rule is based on the law of conservation of flow of charge.

- Kirchhoff's second rule is voltage rule which states that in a closed electrical circuit algebraic sum of emf of cells = algebraic sum of potential drops across resistors.
- $\Sigma E = \Sigma IR$ Second rule is based on the law of conservation of energy.
- Application of Kirchhoff's rules to calculate currents and voltages in simple circuits